FLOW OF A SUPERSONIC JET INTO CHANNELS OF VARIOUS SHAPES

B. A. Balanin

Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 1, pp. 91–97, 1968

UDC 533.601.15:62–225

The author examines the process of flow from one or more supersonic nozzles into a chamber with a diffuser or a wide cylindrical tube. The characteristic regimes are established. An analysis of the experimental data shows that the chamber pressure is at a minimum when critical flow conditions exist at the diffuser outlet.

In certain equipment, such as wind tunnels, supersonic gas ejectors, etc., a supersonic jet ejects air from a certain closed space thereby creating reduced pressure. At the outlet from these systems there is usually a supersonic diffuser or, more commonly, a cylindrical exhaust channel. When air is not supplied from the outside, apart from the nozzle (i. e., when the ejection coefficient is zero), a theoretical analysis is difficult, since it is necessary to take into account the viscosity on the initial section of the jet and consider the process of reconstruction of the free jet into a channel flow.

To establish the corresponding physical flow model, we carried out a large number of experiments [1] on models of the type illustrated in Fig. 1. The experiments were performed with cold air (k = 1.41).

The immediate object of the experiments was to determine the effect of the geometric parameters $\bar{F}_e$, $\bar{l}$, $\bar{L}$, and the M_a number on the dependence of the chamber pressure P_c on the pressure upstream from the nozzle P_0 and ascertain the pressure reduction mechanism.

A typical result is presented in Fig. 1, where P_c is plotted against P_0. These curves take different forms depending on the length of the channel $\bar{l}$. At $\bar{l} < \bar{l}_{opt}$ (curve A on the graph), as P_0 increases the chamber pressure falls slightly (branch I of the curve) until P_0 reaches a certain value. Then, any small increase in P_0 results in a nonsteady process of variation of P_c (i. e., P_c falls continuously at P_0 = const), which continues until (branch II of the curve) P_c reaches a minimum value lying on the branch of the curve characterizing the n_{lim} regime corresponding to the condition P_c/P_0 = const (branch III of the curve). Then the pressure in the chamber becomes proportional to the pressure upstream from the nozzle.

At $\bar{l}_{opt}$ (Fig. 1, curve B), the pressure P_c falls montonically as P_0 increases, reaching a minimum value $(P_c)_{min}$, and the n_{lim} regime sets in. The quantity n_{lim} does not depend on the length of the outlet channel $\bar{l}$. The maximum vacuum also corresponds to $\bar{l} \geq \bar{l}_{opt}$; moreover, when $\bar{l} \geq \bar{l}_{opt}$ the minimum of P_c is reached at the least (as compared with $\bar{l} < \bar{l}_{opt}$) pressure upstream from the nozzle P_{0m}.

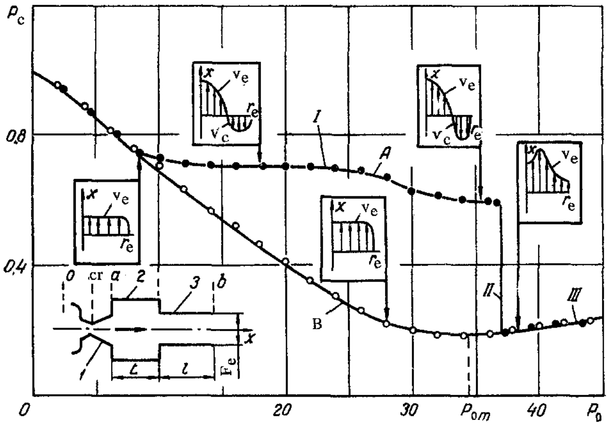

Fig. 1. Chamber pressure P_c, atm abs., as a function of the pressure upstream from the nozzle P_0 atm abs., at M_a = 2.45, $\bar{F}_e$ = 16, $\bar{L}$ = 4.0 (open circles—$\bar{l}$ = 6, solid circles—$\bar{l}$ = 2): 1) nozzle, 2) chamber, 3) outlet channel.

As a result of an analysis of the experimental data we established the physical picture of the processes in the chamber when $\bar{l} < \bar{l}_{opt}$. In this case branch I of the $P_c = f(P_0)$ curve corresponds to small values of n. Moreover, in section b the jet has transverse dimensions much smaller than the clear cross section of the outlet channel, and there is an annular gap between the outer edge of the jet boundary layer and the channel walls.

Owing to turbulent transfer the jet passing through the chamber and the outlet channel picks up and entrains a certain mass of air. In the course of time (at P_0 = const) the pressure in the chamber should fall as a result of mass entrainment, but, as experiments show, it does not depend on time at constant P_0. This indicates the presence of a constant mass influx through the annular gap between the channel walls and the jet to compensate for the additional mass of the jet. Direct experimental investigation confirms the presence of this counterflow, the pressure in it falling from the ambient pressure to the pressure level in the chamber.

As P_0 increases, so does n, and hence the cross section of the jet increases. The annular gap is reduced and the pressure falls, while the velocity in the counterflow v_c increases, since at each instant of time the flow rate in the reverse stream must be equal to the additional mass of the jet. This continues until the velocity of the counterflow in section b becomes equal to the critical velocity $v_c = a_{cr}$. Further increase in P_0 leads to a decrease in the area of the annular gap, but, obviously, the velocity v_c cannot increase any more. Starting from this instant, the flow rate in the reverse direction can no longer compensate the additional mass of the jet, and a nonsteady process of removal of mass from the chamber ensues. The chamber pressure falls at constant P_0. This corresponds to branch II of the curve in Fig. 1. This question was examined in detail in [2].

The transient process ends when the outer edge of the jet begins to touch the walls of the outlet channel.

A further increase in P_0 leads to a reorganization of the flow field in the outlet channel until the streamline in the jet boundary layer on which the velocity is equal to the critical value touches the edge of the channel inlet. This marks the beginning of the n_{lim} regime.

This argument, applicable to the situation when the outlet channel is not sufficiently long ($\bar{l} < \bar{l}_{opt}$) is illustrated in Fig. 1 by the velocity fields at the exit section of the outlet channel.

It is clear from Fig. 1 that on branch I of curve A there are two flows in the exit section: the jet proper and a surrounding annular counterjet with velocity v_c.

At $\bar{l} \geq \bar{l}_{opt}$, the velocity fields corresponding to $P_0 > P_{0m}$ are uniform. Sharp nonuniformity develops only at $P_0 > P_{0m}$.

If $\bar{l} \geq \bar{l}_{opt}$, the jet almost immediately touches the edge of the channel walls; branch II of the curve is very small and is not detected experimentally, although it undoubtedly exists.

The processes are all similar in character when the chamber and outlet channel are replaced simply by a wide tube or when several nozzles are installed in any order in the bottom of a wide tube.

Obviously, then, the characteristic quantities are as follows: the minimum chamber pressure $(P_c)_{min}$, the total pressure upstream from the nozzle P_{0m} at which $(P_c)_{min}$ is reached, and the ratio n_{lim}. Moreover, there is a certain optimal length of the outlet channel $\bar{l}_{opt}$ which ensures maximum pressure reduction at the least pressure upstream from the nozzle. The quantity $\bar{l}_{opt}$ depends only on the M_a number at the nozzle exit and can be found from the empirical formula

$$\bar{l} = 1.78\, M_a.$$

A generalization of the experimental data has shown that when M_a = const the pressure upstream from the nozzle at which the chamber pressure is a minimum (provided that $\bar{l} > \bar{l}_{opt}$) is directly proportional to the relative clear cross section of the outlet channel $\bar{F}_e$. An example of this generalization is shown in Fig. 2, where, as may seen from the graphs, the proportionality factor depends on M_a. Thus, we have experimentally deomonstrated the existence of the condition

$$\frac{\bar{P}_{0m}}{\bar{F}_e} = \text{const when } M_a = \text{const}. \qquad (1)$$

The ratio $\bar{P}_{0m}/\bar{F}_e$ is shown as a function of M_a in Fig. 3, from which it follows that at a given M_a the length of the chamber $\bar{L}$ has almost no effect on $\bar{P}_{0m}/\bar{F}_e$ (at any rate within fairly wide limits, $\bar{L}$ = 0–7).

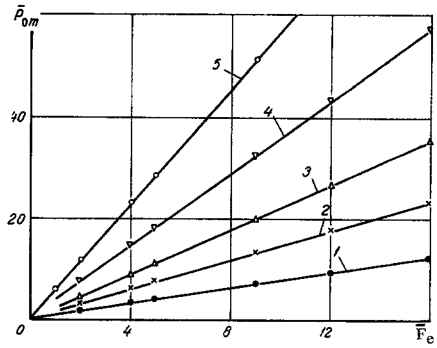

Fig. 2. Pressure P_{0m}, atm abs., at $\bar{l} \geq \bar{l}_{opt}$ as a function of the area of the outlet channel $\bar{F}_e$ at $\bar{L}$ = 4: 1) M_a = = 1.0; 2) 2.02; 3) 2.45; 4) 2.85; 5) 3.37.

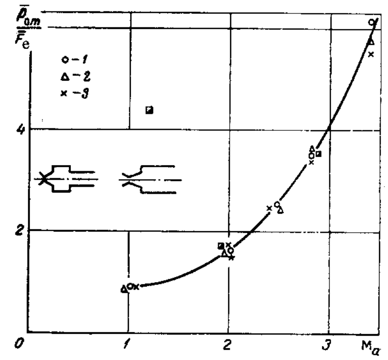

Fig. 3. The ratio $\bar{P}_{0m}/\bar{F}_e$ as a function of the Mach number at the nozzle exit M_a for different models (curve—calculations based on Eq. (4); star—group of four nozzles): 1) $\bar{L}$ = 1.5; 2) 4.0; 3) 7.0.

Essentially, $\bar{P}_{0m}$ is a quantity directly proportional to the flow rate through the nozzle (in supercritical regimes). Consequently, (1) can be represented as follows:

$$\frac{aG_0}{\bar{F}_e} = a\rho_e v_e = \text{const}, \qquad (2)$$

since the flow rate through the nozzle G_0 is equal to the flow rate through the outlet channel G_e. Here, a is a coefficient depending on the gas constant, the adiabatic exponent, and the stagnation temperature upstream from the nozzle. It is easily determined from known relations.

A variety of conditions may exist in the chamber and the outlet channel. At small $\bar{F}_e$ the flow from the nozzle will be characterized by overexpansion and the flow may separate from the nozzle walls; on the other hand, at higher values of $\bar{F}_e$ the underexpanded jet is characterized by a complex system of shock waves that causes an extremely nonuniform velocity field at the inlet to the outlet channel and considerable total pressure losses. Broad variation of the chamber length $\bar{L}$ should also lead to considerable differences in the character of the process. However, as follows from Fig. 2 relation (2) always holds whatever the conditions.

Comparison of Calculated and Experimental Values of P_{0m} and $(P_c)_{min}$

M_a	P_{0m}, atm obs.		ΔP_{0m}, %	$(P_c)_{min}$, atm. abs.		$\Delta(P_c)_{min}$, %
	calc. from (4)	experim.		calc. from (8)	experim.	
2.45	38.1	36	5.5	0.197	0.200	1.5
2.85	31.2	29	7	0.171	0.160	5.5

Measurement of the velocity fields at the exit from the outlet channel in various typical regimes (Fig. 1) shows that in the region where the pressure P_c reaches a minimum (i.e., at $P_0 = P_{0m}$), the nature of the field changes sharply.

It follows that the only point where the flow conditions remain unchanged is the exit section of the outlet channel. It is natural to assume that this point is characterized by a flow with critical parameters when $P_0 = P_{0m}$.

We can now write the equation relating the flow rates through the nozzle and the exit section of the outlet channel for the regime in which P_c reaches its minimum value:

$$\varphi_a m \frac{P_{0m} F_a q(\lambda_a)}{\sqrt{T_0}} = \varphi_e m \frac{P_{em} q(\lambda_{em}) F_e}{\pi(\lambda_{em}) \sqrt{T_{0e}}}. \qquad (3)$$

Experimental investigations of the pressure distribution along the walls of the outlet channel show that in the exit section it is always equal to the ambient pressure P_n. Moreover, for the critical flow regime at the channel exit $q(\lambda_{em}) = 1$. Then, setting $T_0 = T_{0e}$,

$$\frac{\bar{P}_{0m}}{\bar{F}_e} = \frac{P_{0m}}{P_n} \frac{F_a}{F_e} = \frac{\varphi_e}{\varphi_a} \frac{1}{\pi(\lambda_{em}) q(\lambda_a)}. \qquad (4)$$

Here, φ_a and φ_e are the flow coefficients of the nozzle and the outlet channel. Expression (4) corresponds qualitatively to expression (1) obtained on the basis of a generalization of the experimental data. Calculations based on (4) are in good agreement with the experimental data at $\varphi_e/\varphi_a = 0.5$ irrespective of the M_a number and the values of $\bar{F}_e$ and $\bar{L}$.

Varying the parameters over a wide range leads to considerable changes in the total pressure losses due to important changes in the system of shock waves in the jet. The only element to be retained in the experiments is the shape of the exit section of the outlet channel and, obviously, the coefficient φ_e is completely determined by this element. This is reinforced by the fact that in relation to models of other shapes (flow into a wide tube from a single supersonic nozzle or a group of nozzles) the nature of the variation of base pressure as a function of the pressure P_0 is the same as in the basic situation considered above. In these cases, too, the pressure P_{0m} is in good agreement with the calculated value (Fig. 3) at $\varphi_e/\varphi_a = 0.5$.

It should be noted that in examining flow into a chamber or a wide tube from a group of several nozzles, the area of the outlet channel F_e must be related to the total area of the exit sections of all the nozzles.

The value of n_{lim} can be found by the method proposed in [3], but with certain refinements. In [3] it is assumed that the n_{lim} regime develops when the ideal boundary of the jet touches the edge of the inlet to the outlet channel. It appears, however, that closer agreement with experiment can be obtained by assuming that the n_{lim} regime is realized when the streamline in the boundary layer of the jet on which the velocity is equal to the critical value touches the edge of the inlet. The velocity distribution in the boundary layer can be found from the formula

$$\frac{\lambda}{\lambda_t} = \left[1 - \left(\frac{y}{b}\right)^{1.5}\right]^2, \qquad (5)$$

and the condition n_{lim} corresponds to

$$r_e = r_t + y|_{\lambda=1}. \qquad (6)$$

Since the point P_{0m} lies on the branch of the $P_c = f(P_0)$ curve almost corresponding to the n_{lim} regime, knowing n_{lim}, we can easily determine the minimum chamber pressure $(P_c)_{min}$ or the base pressure.

Since

$$n_{lim} = \frac{P_a}{P_c} = \frac{P_0 \pi(\lambda_a)}{P_c}, \qquad (7)$$

we have

$$(P_c)_{min} = \frac{P_{0m} \pi(\lambda_a)}{n_{lim}} = \frac{\bar{\varphi}_e \pi(\lambda_a) \bar{F}_e}{n_{lim} q(\lambda_a)}. \qquad (8)$$

Values of P_{0m} and $(P_c)_{min}$ determined from the relations obtained above, and their experimental values, are presented in the table.

From the table it is clear that on average the discrepancy does not exceed 5%.

The regime corresponding to $(P_c)_{min}$ is the most interesting from the standpoint of applications. Using the above relations we can also find the total pressure losses in this regime:

$$\mu = \frac{P_{0e}}{P_{0m}} = \frac{1}{\bar{\varphi}_e} \frac{q(\lambda_a)}{F_e}. \qquad (9)$$

It should be noted that the calculations and experiments show that in the P_{0m} regime the total pressure losses reach a maximum and at $n = n_{lim}$ retain this value. Calculations based on Eq. (9) agree with the experimental data to within 15%. Thus, it is possible to determine the characteristic quantities n_{lim}, P_{0m}, $(P_c)_{min}$, and μ.

NOTATION

F_e is the clear cross section of the outlet channel; F_a is the area of the nozzle exit section; $\bar{F}_e = F_e/F_a$ is the relative flow section of the outlet channel; L is the length of the chamber; l is the length of the outlet channel; d_a is the diameter of the nozzle exit section; d_e is the diameter of the outlet channel flow section; $\bar{L} = L/d_a$ is the relative length of the chamber; $\bar{l} = l/d_e$ is the relative length of the outlet chamber; $r_e = d_e/2$; $\bar{l}_{opt}$ is the relative length of the outlet chamber ensuring minimum chamber pressure; r_t is the coordinate of the boundary of an ideal jet; b is the thickness of the jet boundary layer; y is the variable coordinate (the y-axis is directed from the boundary of the ideal jet toward the outer edge of the boundary layer); k is the adiabatic exponent; $g = 9.81\ m/sec^2$ is the acceleration of gravity; R is the gas constant; a_{cr} is the critical velocity; M is the Mach number; $\lambda = v/a_{cr}$ is the characteristic velocity; P is the pressure; T is the temperature; ρ is the density; v is the velocity; μ is the total pressure loss.

$$\pi(\lambda) = \left(1 - \frac{k-1}{k+1}\lambda^2\right)^{\frac{k}{k+1}}, \quad q(\lambda) = \lambda\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \times \left(1 - \frac{k-1}{k+1}\lambda^2\right)^{\frac{1}{k-1}},$$

$$m = \sqrt{\frac{kg}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}, \quad a = \frac{\sqrt{T_0}}{mq(\lambda_a)F_a}.$$

Subscripts: 0—stagnation parameters upstream from nozzle; a—parameters at nozzle exit; c—parameters in chamber or wide tube (outside jet); m—parameters in regime corresponding to minimum P_c; n—parameters of ambient medium; e—parameters in exit section of outlet channel; t—parameters at boundary of ideal jet.

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18 September 1967 Leningrad State University